# Sequences & Series

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Name: ________________________

Author(s): D. Loo, S. Lee, E. Ko
1.1 – Arithmetic Sequences

Identify the pattern: \[2, 4, 6, 8, 10, \ldots\]

**Definitions**

In an arithmetic sequence, the number obtained by subtracting any term from the next term is a constant. This constant is the **common difference**, \(d\).

**Arithmetic Sequence:**

A list of numbers where we add the common difference to one term to get the next term in the sequence.

When \(d\) is **positive** , the sequence is **increasing**.

When \(d\) is **negative** , the sequence is **decreasing**.

**Example 1:** For the arithmetic sequence 2, 5, 8, ... determine the sequence up to the 5\(^{th}\) term. Use the pattern to calculate the \(n^{th}\) term of the sequence. \(d = 3\)

<table>
<thead>
<tr>
<th>(n)</th>
<th>(t_n)</th>
<th>(t_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2 + 3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2 + 3(2)</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>2 + 3(3)</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>2 + 3(4)</td>
<td>14</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>(n)</td>
<td>2 + 3((n - 1))</td>
<td>(t_n)</td>
</tr>
</tbody>
</table>

**General Term of an Arithmetic Sequence**

The general term of an arithmetic sequence is given by:

\[t_n = t_1 + d(n - 1)\]

Where:

- \(n\) = \# of terms
- \(t_1\) = value of 1\(^{st}\) term
- \(d\) = common difference

\[t_n = n^{\text{th}} \text{ term}\]
Example 2: In an arithmetic sequence, the 5th term is 56 and the 11th term is 134.

a) List the sequence. Show the first five terms.

b) Write the general term of the sequence.

c) How many terms of the sequence are less than 150?

\[ \begin{align*}
\text{a) } & \quad 4, 17, 30, 43, 56 \\
\end{align*} \]

\[ \begin{align*}
\text{b) } & \quad t_n = t_1 + d(n-1) \\
& \quad t_n = 4 + 13(n-1) \\
& \quad t_n = 4 + 13n - 13 \\
& \quad t_n = 13n - 9 \\
& \text{simplest form} \\
\end{align*} \]

\[ \begin{align*}
\text{c) } & \quad t_n = 13n - 9 \\
& \quad 150 = 13n - 9 \\
& \quad +9 +9 \\
& \quad 159 = 13n \\
& \quad 13 \quad 13 \\
& \quad n = 12.2307... \\
& \text{12 terms less than 150} \\
& \text{n must be a natural number} \\
& (1, 2, 3, 4, ...) \\
\end{align*} \]
Example 3: In the arithmetic sequence 8, 14, 20, 26, … determine each term
a) \( t_n \)
\[
\begin{align*}
  bn &= b_1 + d(n-1) \\
  bn &= 8 + 6(n-1) \\
  bn &= 8 + 6n - 6 \\
  bn &= 6n + 2
\end{align*}
\]
b) \( t_{20} \)
\[
\begin{align*}
  bn &= 6n + 2 \\
  b_{20} &= 6(20) + 2 \\
  b_{20} &= 122
\end{align*}
\]

Example 4: A sum of $83 was deposited in a bank on January 1st. A sum of $20 is deposited in the bank on the 12th day of each month. Suppose this pattern continues, how much will be in the bank on September 1st?

<table>
<thead>
<tr>
<th>J. 1st</th>
<th>J. 12th</th>
<th>F. 12th</th>
<th>...</th>
<th>A. 12th</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>103</td>
<td>123</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Balance = $83 + 20(8) = \frac{83 + 160}{2} = \$243$

Example 5: Determine the number of terms in the arithmetic sequence –12, –6, 0, … , 72

\[
\begin{align*}
  n &= ? \\
  d &= 6 \\
  b_1 &= -12 \\
  t_n &= 72 \\
  bn &= b_1 + d(n-1) \\
  72 &= -12 + 6(n-1) \\
  +12 &+12 \\
  84 &= 6(n-1) \\
  \frac{84}{6} &= \frac{6(n-1)}{6} \\
  14 &= n-1 \\
  +1 &+1 \\
  n &= 15
\end{align*}
\]

Assignment
Page 16, #1ac, 2b, 5bc, 6c, 7, 9-11, 14, 16ab, 17, 25
# 13.

\[ f.1 \quad f.2 \quad f.3 \]

\[ \begin{align*}
9) & \quad P = 10 + 6(n-1) \\
b) & \quad P = 10 + 6(n-1) \\
& \quad = 58 \\
\end{align*} \]

\[ \begin{align*}
10) & \quad 76 = 6 + 6(n-1) \\
& \quad 66 = 6(n-1) \\
& \quad 11 = n-1 \\
& \quad n = 12 \\
\end{align*} \]
1.2 – Arithmetic Series

Recall: 2, 5, 8, 11, 14, 17 ← arithmetic sequence

\[ 2 + 5 + 8 + 11 + 14 + 17 \] ← arithmetic Series

Definitions

Arithmetic Series: the sum of the terms of a given arithmetic sequence.

This method can be used to calculate the sum of any number of terms of an arithmetic series.

Deriving the formula for the sum of the first \( n \) terms of the general arithmetic series:

\[
S_n = t_1 + (t_1+d) + (t_1+2d) + \ldots + [t_1+d(n-1)]
\]

\[
2S_n = [2t_1+d(n-1)] + [2t_1+d(n-2)] + \ldots + [2t_1+d(n-1)]
\]

\[
2S_n = 2t_1 + d(n-1) + 2t_1 + d(n-2) + \ldots + 2t_1 + d(n-1)
\]

\[
\frac{2S_n}{2} = n \left[ 2t_1 + d(n-1) \right]
\]

\[
S_n = \frac{n\left[ 2t_1 + d(n-1) \right]}{2}
\]

\[
S_n = \frac{n\left( t_1 + t_n \right)}{2}
\]

The Sum of \( n \) terms of an Arithmetic Series

\[
S_n = \frac{n\left[ 2t_1 + d(n-1) \right]}{2} \quad \text{or} \quad S_n = \frac{n\left( t_1 + t_n \right)}{2}
\]

Where: \( n = \# \text{ of terms} \) \hspace{1cm} \( d = \text{common difference} \) \hspace{1cm} \( S_n = \text{sum of the first } n \text{ terms.} \)
Example 1: Determine the sum of the first 25 terms of the arithmetic series
\[ 2 + 8 + 14 + \ldots \]
\[
S_{25} = \frac{n[2t_1 + d(n-1)]}{2}
\]
\[
S_{25} = 25 \left[ \frac{2(2) + 6(25-1)}{2} \right]
\]
\[
S_{25} = \frac{25[140]}{2} = 1850
\]

Example 2: Determine the sum of the following arithmetic series.
\[ 2 + 6 + 10 + \ldots + 122 \]
\[
t_1 = 2, \quad t_n = 122
\]
\[
n = ?
\]
\[
t_n = t_1 + d(n-1)
\]
\[
122 = 2 + 4(n-1)
\]
\[
120 = 4(n-1)
\]
\[
30 = n-1
\]
\[
31 = n
\]
\[
S_n = \frac{n(t_1 + t_n)}{2}
\]
\[
S_{31} = \frac{31(2 + 122)}{2} = 1922
\]

Example 3: Determine \( S_{42} \) given the following series: \[ 1 + 3.5 + 6 + 8.5 + \ldots \]
\[
d = 3.5 - 1, \quad n = 42
\]
\[
d = 2.5, \quad t_1 = 1
\]
\[
S_n = \frac{n[2t_1 + d(n-1)]}{2}
\]
\[
S_{42} = \frac{42[2(1) + 2.5(42-1)]}{2}
\]
\[
S_{42} = \frac{42(104.5)}{2} = 2194.5
\]
Example 4: Determine the value of the first term.
\[ d = 4 \quad n = 20 \quad S_{20} = 1380 \]

\[
S_n = \frac{n(2t_1 + d(n-1))}{2}
\]

\[
S_{20} = \frac{20(2t_1 + 4(20-1))}{2}
\]

\[
2(1380) = \left(\frac{20(2t_1 + 76)}{2}\right)
\]

\[
2760 = 20 \cdot \frac{2t_1 + 76}{2}
\]

\[
138 = 2t_1 + 76
\]

\[-76 = -76
\]

\[
t_1 = 31
\]

Example 5: Nathan Gerelus is a Manitoba farmer preparing to harvest his field of wheat. Nathan begins harvesting the crop at 11:00 a.m., after the morning dew has evaporated. By the end of the first hour he harvests 240 bushels of wheat. Nathan challenges himself to increase the number of bushels harvested by the end of each hour. Suppose that his increase produces an arithmetic series where Nathan harvests 250 bushels in the second hour, 260 bushels in the third hour, and so on.

a) Write the series that would illustrate the amount of wheat that Nathan has harvested by the end of the seventh hour.

\[ 240 + 250 + 260 + 270 + 280 + 290 + 300 \]

b) Write the general sum formula that represents the number of bushels of wheat that Nathan took off the field by the end of the \( n \)th hour.

\[ S_n = \frac{10n^2 + 470n}{2} \]

\[ S_n = 5n^2 + 235n \]

\[ S_7 = 5(7)^2 + 235(7) \]

\[ S_7 = 1890 \]

c) Determine the total number of bushels harvested by the end of the seventh hour.

d) State any assumptions that we can make.
a) He works at the same rate every hour.
1.3 – Geometric Sequences

Identify the pattern: $1, 4, 16, 64, \ldots$

$\times 4 \quad \times 4 \quad \times 4$

Definitions
In a geometric sequence, the ratio formed by dividing any term by the preceding term is a constant. This constant is the **common ratio** ($r$).

Geometric Sequence: a sequence of numbers where we multiply a term by the common ratio to get the next term.

Example 1: For the geometric sequence 2, 6, 18, … determine the sequence up to the 5$^{\text{th}}$ term. Use the pattern to calculate the $n^{\text{th}}$ term of the sequence.

<table>
<thead>
<tr>
<th>$t_1$</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_2$</td>
<td>$2 \times 3$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>$2 \times (3)^2$</td>
</tr>
<tr>
<td>$t_4$</td>
<td>$2 \times (3)^3$</td>
</tr>
<tr>
<td>$t_5$</td>
<td>$2 \times (3)^4$</td>
</tr>
<tr>
<td>$t_n$</td>
<td>$2 \times (3)^{n-1}$</td>
</tr>
</tbody>
</table>

General Term of a Geometric Sequence

The general term of a geometric sequence is given by:

$$t_n = t_1 r^{n-1}$$

Where: $t_1 = 1^{\text{st}}$ term

$r =$ common ratio

$n =$ # of terms

$\frac{6}{2} = 3$
Example 2: For the geometric sequence; \(-16 \,000, \, 8000, \, -4000, \, 2000, \ldots\) determine the 9th term.

\[ r = \frac{8000}{-16000} = -0.5 \]

\[ t_9 = t_1 r^{n-1} \]

\[ t_9 = -16000(-0.5)^9 \]

\[ t_9 = -62.5 \]

Some examples of geometric sequences:

The sequence 4, 12, 36, 108, \ldots is an \underline{infinite} geometric sequence because it continues forever.

The sequence 4, 12, 36, 108 is a \underline{finite} geometric sequence because the sequence is limited to a fixed number of terms.

- This is an increasing geometric sequence because the terms are increasing: 2, 10, 50, 250, 1250, \ldots. The sequence is \underline{divergent} because the terms do not approach a constant value.

- This is a geometric sequence that neither increases, nor decreases because consecutive terms have numerically greater values and different signs: 1, -2, 4, -8, 16, \ldots. The sequence is \underline{divergent} because the terms do not approach a constant value.

- This is a decreasing geometric sequence because the terms are decreasing: \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\) The sequence is \underline{convergent} because the terms approach a constant value of 0.

Example 3: Pablo purchases a painting valued at $8500. Art appraisers estimate that the value of the painting will appreciate at a rate of 12% per year. How much will the painting be worth after 15 years?

\[ r = 1.12 \]

\[ \text{Value in } 15 \text{ years} = 8500 (1.12)^{15} \]

\[ \text{Value in } 15 \text{ years} = \# \ 46525.31 \]
Example 4: In a geometric sequence, the third term is 567 and the seventh term is 3,720,087. Determine the value of $t_1$. Is the sequence convergent or divergent?

\[ t_1 = t_3 \cdot r^4 \]
\[ 3,720,087 = 567 \cdot r^4 \]
\[ \frac{3,720,087}{567} = r^4 \]
\[ \sqrt[4]{567} = r \]
\[ \pm 9 = r \]

Determing $t_3$:
\[ t_3 = t_1 \cdot r^2 \]
\[ 567 = (t_1 \cdot 9)^2 \]
\[ 567 = 81t_1 \]
\[ t_1 = 7 \]

Example 5: The number of an ant colony triples every year. There are currently about 500 ants in the colony.

a) How many ants will there be after 4 years?  
b) Show the results on a graph.  
c) Write an expression to represent the number of ants after $n$ years.

a) Currently after 1 year after 2 years after 3 years after 4 years

\[ \begin{array}{ccc}
500 & 1500 & 4500 \\
\text{Ants in 4 years} & = & 4500 \\
\text{Ants in } n = 500(3)^n \\
\text{Ants after } & \text{apply } r \\
\text{4 years} & = & 4500 \\
\end{array} \]

b) Show the results on a graph.

\[ t_n = 500(3)^n \]

or

\[ t_n = 1500(3)^{n-1} \]

**Given the value of the common ratio $r$, how do you know if a geometric sequence is convergent or divergent?**

Assignment
Page 39, #1bcf, 3d, 4, 5c, 7, 9, 10, 12, 20
1.4 – Geometric Series

Recall:

\[ 2, 4, 8, 16, 32, 64 \quad \text{geometric sequence} \]
\[ 2 + 4 + 8 + 16 + 32 + 64 \quad \text{geometric series} \]

Definitions

Geometric Series: the sum of the terms of a given geometric sequence.

This method can be used to calculate the sum of any number of terms of a geometric series.

Deriving the formula for the sum of the first \( n \) terms of the general geometric series:

Geometric Sequence Formula:

\[ t_n = a \cdot r^{n-1} \]

Geometric Series:

\[ S_n = t_1 + t_1r + t_1r^2 + \ldots + t_1r^{n-1} \]

\[ -r \cdot S_n = -t_1r + t_1r^2 + \ldots + t_1r^{n-1} + t_1r^n \]

\[ S_n - rS_n = t_1 - t_1r^n \]

\[ S_n(1-r) = t_1(1-r^n) \]

\[ S_n = \frac{t_1(1-r^n)}{1-r} \]

The Sum of \( n \) terms of a Geometric Series

\[ S_n = \frac{t_1(1-r^n)}{1-r} \quad \text{or} \quad S_n = \frac{t_1(r^n-1)}{r-1} \]

Where:

\( n \) = number of terms
\( t_1 \) = first term
\( r \) = common ratio
\( S_n \) = sum of the first \( n \) terms
Example 1: Determine the sum of:

a) the first 9 terms of the geometric series \(2 + 6 + 18 + 54 + \ldots\)

\[
S_n = \frac{t_1 (1 - r^n)}{1 - r} \quad S_9 = \frac{2 (1 - 3^9)}{1 - 3} = 19682
\]

b) the first 9 terms of the geometric series \(2 - 6 + 18 - 54 + \ldots\)

\[
S_n = \frac{t_1 (1 - r^n)}{1 - r} \quad S_9 = \frac{2 [1 - (-3)^9]}{1 - (-3)} = 9842
\]

c) the first 10 terms of the geometric series \(6 + 3 + 1.5 + \ldots\)

\[
r = \frac{3}{6} = 0.5
\]

\[
S_{10} = \frac{6 (1 - 0.5^{10})}{1 - 0.5} = 11.99
\]

Example 2: Find the sum of the geometric series: \(1 + 5 + 25 + \ldots + 3125\).

Method #1

\[
t_1 = 1 \quad t_n = 3125
\]

\[
r = 5 \quad n = ?
\]

\[
t_n = t_1 r^{n-1}
\]

\[
3125 = 1 (5)^{n-1}
\]

\[
5 = 5^{n-1}
\]

\[
5 = 5^{n-1}
\]

\[
S_n = \frac{t_1 (1 - r^n)}{1 - r}
\]

\[
S_6 = \frac{1 (1 - 5^6)}{1 - 5}
\]

\[
S_6 = 3906
\]
Example 3: If you are paid $0.01 on the first day of work, $0.02 on the second day, $0.04 on the third day and so on… how much money would you have after working 30 days in a row at this rate? Would you take this job?

\[ t_1 = 0.01 \hspace{1cm} r = 2 \hspace{1cm} n = 30 \]

\[ S_n = \frac{t_1(1 - r^n)}{1 - r} \]

\[ S_{30} = \frac{0.01(1 - 2^{30})}{1 - 2} \]

\[ S_{30} \approx 10,737,418.23 \]

Yes please!
Example 4: Given that $S_1 = 8$ and $S_2 = 20$, determine the value of $S_6$.

\[ S_1 = t_1 \]
\[ t_1 = 8 \]
\[ n = 6 \]
\[ S_2 = t_1 + t_2 \]
\[ 20 = 8 + t_2 \]
\[ t_2 = 12 \]

\[ r = \frac{12}{8} = 1.5 \]

\[ S_n = \frac{t_1(1-r^n)}{1-r} \]
\[ S_6 = \frac{8(1-1.5^6)}{1-1.5} \]

\[ S_6 = 166.25 \]

Assignment
Page 53, #1, 2b, 3b, 4a, 6-7, 9-11, 13, 16, 20
1.5 – Infinite Geometric Series

Recall: Given the value of the common ratio \( r \), how do you know if a geometric sequence is convergent or divergent?

\[
\text{if } -1 < r < 1 \quad \text{, } \quad r \neq 0
\]

**Definitions**

**Convergent series:** A series with an infinite number of terms, in which the sequence of partial sums approaches a constant value.

**Divergent series:** A series with an infinite number of terms, in which the sequence of partial sums does not approach a constant value.

### Infinite Geometric Series

Let’s calculate the partial sums and predict if there is a sum at infinity (the sum of all the infinite number of terms)

\[
\begin{align*}
\text{(a) } & \quad 5 + 10 + 20 + 40 + 80 + \ldots & \quad \text{(b) } & \quad \frac{5}{4} + \frac{5}{16} + \frac{5}{64} + \frac{5}{256} + \ldots \\
S_1 &= 5 & S_1 &= 5 \\
S_2 &= 5 + 10 = 15 & S_2 &= 5 + \frac{5}{4} = 6.25 \\
S_3 &= 5 + 10 + 20 = 35 & S_3 &= 5 + \frac{5}{4} + \frac{5}{16} = 6.5625 \\
S_4 &= 5 + 10 + 20 + 40 = 75 & S_4 &= 5 + \frac{5}{4} + \frac{5}{16} + \frac{5}{64} = 6.640625 \\
S_5 &= 5 + 10 + 20 + 40 + 80 = 155 & S_5 &= 6.66015625 \\
S_\infty &= & S_\infty &= 6.6666666\ldots
\end{align*}
\]
The values of $S_n$ for series (a) becomes larger and larger quickly, but what about series (b)? Let’s analyze the expression for $S_n$ for the following general geometric series:

$$t_1 + t_1r + t_1r^2 + t_1r^3 + \ldots$$

What will happen to this expression as the number of terms in our series, $n$, increases and approaches infinity?

As $n$ becomes large… the value of $r^n$ becomes very small….

$$n \rightarrow \infty \quad r^n \rightarrow 0$$

$$S_n = \frac{t_1(1-r^n)}{1-r} \rightarrow S_\infty = \frac{t_1(1-r^\infty)}{1-r} \rightarrow S_\infty = \frac{t_1}{1-r}$$

**The Sum of an Infinite Geometric Series**

$$S_\infty = \frac{t_1}{1-r} \quad ; \quad -1 < r < 1 \quad r \neq 0$$

Where:

$S_\infty$ = sum of infinite series

$\ t_1$ = 1st term

$r$ = common ratio

**Example 1:** Consider the infinite geometric series $50 - 25 + 12.5 - 6.25 + \ldots$

a) Does the sum of the series exist?

$$r = \frac{-25}{50} \rightarrow r = -\frac{1}{2}$$

Yes, $b/c$ $r$ is between $-1$ and $1$, the series is convergent therefore has an infinite sum.

b) Determine the sum to infinity.

$$S_\infty = \frac{\frac{1}{2}}{1-\left(-\frac{1}{2}\right)} = \frac{\frac{50}{1-(\frac{1}{2})}}{\frac{3}{2}} = \frac{50 \times \frac{2}{3}}{\frac{3}{2}} = \frac{100}{3}$$
Example 2: Which infinite geometric series has a sum? What is the sum?

a) $3 - 6 + 12 - 24 + \ldots$
   
   $r = -\frac{6}{3} = -2$

   Infinite sum DNE (does not exist)

b) $5 \cdot \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \ldots$

   $r = -\frac{\frac{5}{3}}{\frac{5}{9}} = -\frac{3}{5}$

   $S_\infty = \frac{\frac{51}{5}}{1 - \frac{3}{5}} = \frac{\frac{15}{4}}{\frac{2}{5}} = \frac{15}{4}$

Example 3: A geometric series is $1 + 4x + 16x^2 + 48x^3 + \ldots$

a) Write an expression for the sum to $n$ terms, in terms of $x$.

   $S_n = \frac{1 - (4x)^n}{1 - 4x}$

b) For what values of $x$ does the series have a sum to infinity?

   $-1 < r < 1$

   $-1 < 4x < 1$

   $-\frac{1}{4} < x < \frac{1}{4}$

Example 4: If a geometric series has an infinite sum of $\frac{8}{21}$, determine the first term if the common ratio is $-\frac{3}{4}$.

   $t_1 = ?$

   $S_\infty = \frac{8}{21}$

   $r = -\frac{3}{4}$

   $S_\infty = \frac{t_1}{1 - r}$

   $\frac{8}{21} = \frac{t_1}{1 - (-\frac{3}{4})}$

   $\frac{7}{4} \left( \frac{8}{21} \right) = \left( \frac{t_1}{\frac{7}{4}} \right)^{\frac{7}{4}}$

   $t_1 = \frac{3}{3}$
Example 5: In its first month, an oil well near Virden, Manitoba produced 24 000 barrels of crude. Every month after that, the well produced 6% less oil than the previous month’s production.

a) If this trend continued, what would be the lifetime production of this well?

\[
S_n = \frac{t_1}{1 - r} = \frac{24000}{1 - 0.94} = 400 000 \text{ barrels}
\]

b) What assumption are you making? Is your assumption reasonable?

Oil is reduced at a constant rate.

No

Assignment
Page 63, #1, 2ab, 5-7, 10-12, 14, 16, 18
Ch. 1 Review H/W
p. 66
# 1-2, 3ab, 4a, 5-6, 7bc, 8-11, 12ac, 13-14, 16,
17ab, 18, 19, 20ab, 21ad, 22

Review

# 8

\[ S_{12} = 186 \]

\[ t_{20} = 83 \]

\[ a_3 = t_1 + ad_1 \]

\[ a_3 = t_1 + 19d \]

\[ t_1 = a_3 - 19d \]

\[ t_1 = a_3 - 19(5) \]

\[ b_1 = -12 \]

\[ S_{40} = \frac{40}{2} \left[ 2(-12) + 5(40-1) \right] \]

\[ S_{40} = 3420 \]
# 13. 5000 bacteria

\[ 5000 (1.08)^1 \]
\[ 5000 (1.08)^2 \]
\[ 5000 (1.08)^5 \]

Increases by 8% every hour

\[ b_n = 5000 (1.08)^{n-1} \]

\[ b_6 = 5000 (1.08)^5 \]

# 14.

<table>
<thead>
<tr>
<th>Original</th>
<th>Stage 1</th>
<th>Stage 2</th>
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<tbody>
<tr>
<td>radius = 81</td>
<td>radius = 27</td>
<td>radius = 9</td>
</tr>
</tbody>
</table>

\[ r = \frac{1}{3} \]

\[ C = 2\pi r \]

\[ C \rightarrow 162\pi \]
\[ 54\pi \]
\[ 18\pi \]

\[ r = \frac{54\pi}{162\pi} = \frac{1}{3} \]

\[ 162\pi \left( \frac{1}{3} \right)^4 \]
\[ 2\left( 162\pi \left( \frac{1}{3} \right)^4 \right) \]

\[ 2\pi \]